



# SURVEY ON ROBUST CONTROL APPLICATIONS IN INVERTED PENDULUM SYSTEMS

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**Abstract - The Inverted Pendulum (IP) systems are the benchmark problems in control literature. The general control strategy is the feedback control system, which can be used to measure continuously the angle of the pendulum, thereby commanding the cart to move to and fro so that the pendulum will be balanced in the upright position. Though there are control simulations with experimentation available, about half century back, the robust control studies with IP have gained remarkable attention only in the last two or three decades. The robust techniques like  $H_\infty$ ,  $\mu$ -Synthesis, SMC, LTR, Lyapunov approach etc. are mentioned here. Control experimentation with IP related studies based on robust point of view is very essential since the growing importance of robust techniques in control system community. The proposed paper is a brief survey of the so called literature mentioned in the past three or four years.**

**Key words:** Inverted Pendulum, balancing, robust control,  $H_\infty$ , stabilization

## I. INTRODUCTION

The Inverted Pendulum System (IP) control is a standard benchmark problem in many control applications. When the centre of mass of the pendulum is situated above its pivot point, it is called an inverted pendulum. If an extra support is not provided, this unstable system will fall. The rocket or missile guidance technology is related to IP systems

in some way, where the centre of drag is located in front of the centre of gravity and it ultimately leads to aerodynamic instability. An upturned broomstick at the tip of one's forefinger is a simple demonstration this experiment.

The validation of many novel control theories are investigated through Inverted Pendulum system experimentation. The robust control techniques are invariably applied in Inverted Pendulum System starting from its inception from the beginning of 1980's. The novel robust control based strategies, when implemented in Inverted Pendulum Systems and its derivatives, produces versatile results especially during the last three to four years. This paper analyzes the noteworthy results during this period in the IP based system and point out the significant results out of it. The mathematical modeling of the Inverted Pendulum System is expressed in an approximate normal form with the help of lie derivatives in [1]. The conventional, integral and super twisting sliding mode robust control strategies are used for its implementation. A combination of classical PID, fuzzy and  $H_\infty$  robust control methods results in a new  $H_\infty$  – fuzzy – PID controller, which could produce transient and steady state up gradation in the Inverted Pendulum model proposed in [2]. The classical PID, when compared with cascade controllers, produces distinct performance measures which are indicative of their control ability to maintain the pendulum in upright position [3]. The desired rise time and settling time behavior are more pronounced in the cascade.

## II. SYSTEM DYNAMICS

An inverted pole, hinged on the top of the cart of mass  $M$ , having two degrees of freedom is free to move in a vertical plane whereas the cart which is having only one degree of freedom, is able to move along the track in the horizontal direction only as shown in figure.1. The system dynamics is based on a few valid assumptions of which the equilibrium starting state and the small pendulum angle movement in terms of a few degrees are more relevant. The system dynamics analysis is based on Newton's second law of motion with suitable free body diagrams [2-6]. Its analytical discussion is not going to discuss in this article.



Fig. 1

## III. MAJOR MILESTONES

The double inverted pendulum, as an advanced variant of the conventional IP model, is also subjected to control law experiments.  $H_\infty$  robust experimentation in the proposed double inverted pendulum model accompanied with the  $\mu$ -synthesis controller is given prior focus in [4] where the superiority of  $\mu$ -synthesis controller is established over  $H_\infty$  in achieving robust performance against parametric uncertainties in pendulum moment of inertia and in the friction coefficient of hinges and cart. Euler – Lagrangian method is used for the system dynamic modeling here. The  $H_\infty$  theory, in a constraint manner is applied in [5] in which the Kalman filter and loop transfer recovery (LTR) techniques are subsequently integrated in the implementation of control law in a mobile inverted pendulum so that the constraint satisfaction is guaranteed with sufficient stabilization.

The parametric uncertainty, created by the varied pendulum mass is handled with a novel robust LQR based adaptive neural fuzzy inference System controller in [6] where the robust performance is

ensured in a satisfactory manner. The  $H_\infty$  loop shaping techniques play a big role in inverted pendulum system tracking. The authors in [7] propose an  $H_\infty$  control design which is synthesized using the loop shaping principles. The singular value response tracking had been done accordingly.

An adaptive neural network controller is proposed in [8] for stabilization control and swing up motion of an under actuated rotary inverted pendulum system. A trajectory planning mechanism is incorporated with this new adaptive neural network controller.

The fractional order control principles, when implemented in IP platforms, came up with accurate regulation in pendulum angle control within inner primary loop and in cart position within outer secondary loop [9]. The robust and optimal control principles are combined to get the fractional order  $PI^\lambda D^\mu$  control. But in [10], the altitude stabilization is improved with an intelligently optimized self-tuning fractional order control scheme. Its robustness is validated with a PSO – based fixed gain dual FPD control scheme.

The two well known control laws  $H_\infty$  and backstepping are applied to stabilize the tracking error for a wheeled inverted pendulum system in [11] where the control laws were based on Lyapunov's direct technique. The system proposed here is under actuated external disturbance prone non-linear model.

The feedback linearization control theory, even though it seems to be somewhat not current, when combined with online trajectory planning based on adaptive frequency oscillators for 3 – dot can be very useful in applying on an under actuated pendulum like robot [12]. Since the Inverted Pendulum is a typical example of a real world unstable control system challenge, the particle swarm optimization based neural network controller is practiced in [13]. This improved PSO technique can alleviate a lot of unstable system challenges presented here.

A general Lyapunov function based Uncertain Furuta Pendulum [17] and mixed  $H_\infty$  Controller based rotary double inverted pendulum [15] are two major works which utilize the novel robust control strategies. The linear and non-linear switched controls along with its stability analysis and implementation with IP system is a noteworthy work based on its novelty and innovativeness [16]. The robust generalized dynamic inversion control technique can be added to this category where in [17], a rotary inverted pendulum is subjected with this control technique.

When the medium is water, the system analysis is more complicated. The upright stability of



an inverted pendulum in an underwater environment has been analyzed in [18] where the system is conceptualized for future wave energy harvesting. The state feedback fuzzy control approach is a novel concept which could be used for creating a fuzzy  $H_\infty$  controller in [19] based on simple convex polytopic transformation. State feedback based dynamic Lagrangian conceptualization is proposed for its implementation in the double IP system. With an observer based state feedback concept, Siliet and Yaman could make an on – off type cold gas thrusters controlled rotary inverted pendulum [20]. In [21] a compound disturbance observer is proposed by Trans et.al.

Implementation of an Integral Sliding Mode Controller (ISMC) for Spatial IP Stabilization will eliminate the non-robust reachability phase associated with SMC. The work in [22] is an excellent result based on this fact. A combined SMC and LQR based control system application in [23] results in 29% reduction in root mean square error in pendulum angular control. The sliding mode control based works are continued during this time. A robust sliding mode control is also designed for an under actuated cart-type IP with time-varying uncertainties [24].

Using the possibilities in Internet of Things (IoT), Maneetham and Sutyasadi could design an IP system control with LQR control [25]. The IoT communication developed can monitor the performance results for this feedback controlled system. The IP system control performance investigation was illustrated in Simulink by Jibril [26] where the robust control tools are familiarized by the author, such as those associated with mixed  $H_\infty$  loop shaping control. Reduction in rise time, settling time and percentage overshoot behavior is accomplished over the loop shaping design using Glover Mc Farlane method which is a noticeable event.

When, if one considers the non-linear unstable servo-driven IP system, the placing of closed loop poles using LMI techniques provides satisfactory results despite the parameter variations and external noise signals as in [27].

This year witnesses the experimentation and implementation of robust control theory via floating air levitation and balancing rotary inverted pendulum [28]. Its objective is to stabilize the inverted pendulum upright. PLC based implementation made it possible within this system. The robustness in SMC control techniques in [29-30] can be adapted to IP related works in future.

#### IV. CONCLUSION

In this paper, a brief survey of the inverted pendulum control systems with special focus in application of robust control techniques was done. Since the recent historical evolution of robust strategies is in focus, system dynamics is not elaborated in detail. The robust control applications in Inverted Pendulum Systems in the last three to four years particularly are mentioned in this paper and the highlights of their major works are addressed along with. Since the IP Systems are ideal benchmark problems, the experimentation of novel robust control techniques are easier to validate. This new literature survey methodology will help a robust control engineer to validate his works and to impart motivation in doing further research.

#### V. REFERENCES

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